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Optimal Public Sector Premium, Talent Misallocation, and Aggregate Productivity

Evidence from the Middle East and North Africa

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Abstract

This paper develops a tractable general equilibrium model to quantify the aggregate productivity gains from adjusting the public sector premium and the size of the public sector to their optimal levels. In the framework, the optimal size of the public sector is contingent on the efficiency level of public goods in increasing the productivity of the private sector. The model also incorporates an endogenous decision between market and non-market activities for women. The model is calibrated using data from the Arab Republic of Egypt, a country that exhibits a disproportionate share of workers, and women especially, in the public sector. The findings show that, under a conservative value for the efficiency of the public sector, aligning the public sector premium with its optimal level, thus lowering the share of employment in the public sector, results in aggregate efficiency gains of 12 percent for output per worker and 8 percent for total factor productivity. For lower values of the elasticity of private output to public goods, the productivity gains are almost twice as large. The optimal premium is positive for women and approaches zero for men, preventing a shift of mid-high-level skilled women from the public sector to non-market activities and also a contraction of the male entrepreneurial sector. Notably, a reduced female public sector premium fosters greater female labor force participation in market activities through an expansion of the female entrepreneurial sector, which increases the demand for production labor and drives wages up.

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Optimal Public Sector Premium, Talent Misallocation, and Aggregate Productivity: Evidence from the Middle East and North Africa*

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1 Introduction

In many countries, the structure of wages and labor law legislation differs between public and private sector employees. For example, vast evidence shows that a public-private wage gap exists (Tansel, 2005; Postel-Vinay and Turon, 2007; Depalo et al., 2015; Calvalcanti and Rodrigues do Santos, 2021; Abdallah et al. 2023; among others). In some cases, the pension system also differs between the private and public sectors (Calvalcanti and Rodrigues do Santos, 2021). Moreover, many countries have labor legislation that translates into more secure jobs in the government than in private firms (Clark and Postel-Vinay, 2009; OECD, 2011; Piketty, 2014; He et al., 2018; Calvalcanti and Rodrigues do Santos, 2021).[1] These facts raise some questions: Is the public sector too large? Are public sector workers being overpaid? By how much? What are the efficiency gains from adjusting the public sector premium to its optimal level? What is the optimal size of the public sector?

This paper develops a tractable model to quantify the aggregate productivity gains from adjusting the public sector premium and the size of the public sector to their optimal levels. Our economy model delivers the optimal public sector premium, and thus the optimal size of the public sector, as a function of the efficiency level of public spending. To account for potential non-pecuniary benefits that might attract workers to the public sector (such as more job stability, fewer hours worked, prestige), we do not refer to the premium as a wage premium. Key aspects of our model are the existence of a home-production sector and the demand for managerial talent in the public sector.[2] Our paper then helps shed light on the trade-offs involved in the public sector demand for talent, and how the public sector premium affects aggregate output and productivity.

Several examples can be found of managerial skills applied in the context of the public sector to effectively lead organizations and serve the needs of the population. Some of these skills include effective communication, strategic planning, budgeting and financial management, policy development and implementation, team leadership, conflict resolution, adaptability and flexibility, among others. Public sector managers endowed with more of these skills produce more of a given public good. In contrast, the lack of managerial skills in the public sector

[1] We use the terms public sector, public firm, and government interchangeably.

[2] For background literature on the home-production sector, see Becker (1964), Becker (1965), Benhabib et al. (1991), Aguiar and Hurst (2007), Been et al. (2020), among others.

translates into a very poor production of public goods. Therefore, the public sector needs to attract talented managers. However, these type of agents can also use their managerial skills (effective communication, strategic planning, team leadership, etc.) to run businesses privately. In other words, managerial skills are also valuable in the private sector to run private businesses. Then, the government must pay potential public sector managers a wage high enough to compensate them for the potential earnings they could obtain running their own firms. These potential earnings would be well above the production labor wage for individuals endowed with more managerial skills. Consequently, a premium arises in the public sector to incentivize some individuals with managerial talent to leave their private activities and accept managerial roles in the public sector. Because managerial talent can also be attracted to the public sector through non-pecuniary benefits, we define the *public sector premium* as the package of monetized or pecuniary benefits received by public sector managers relative to the wage paid to production workers.

In our model, public goods enhance the productivity of private activities (e.g., public investment in infrastructure) and talent must be allocated between the public and private sectors. A low public sector premium attracts less talent to the government, which lowers the amount of public goods that boost private output, but expands the private sector and results in a larger number of firms. On the other hand, a high public sector premium attracts more talent to the public sector but shrinks the private sector. Fewer firms would produce but with a larger amount of the public good. This trade-off suggests the existence of an optimal premium and an optimal size for the public sector. Importantly, this optimal premium depends on how intensively the public good enhances private sector productivity. The higher the elasticity of private sector output to the public good, the higher the optimal premium and the larger the size of the public sector.

We consider an economy populated by women and men, each endowed with units of managerial skill. The economy consists of three sectors: the private sector, the public or government sector, and the home production sector (only among women). Individuals make decisions regarding employment in either the private or public sector. Additionally, women have the option to engage in home production. In the private sector, individuals may choose to operate their own businesses or work as production workers for a wage. In the public sector, individuals are employed for a wage, either as production workers or managers. Managerial roles in the public sector receive a gender-specific premium. In equilibrium, the most skilled women operate businesses in the private sector, mid-high skilled women transition between managerial roles in the public sector and home production, mid-low skilled women engage in production at home, while the least skilled join the production labor force. Regarding men, the lowest-skilled individuals work as production workers, some with mid-skills assume managerial roles in the public sector, and the most skilled run private businesses.

We calibrate the model to match data from the Arab Republic of Egypt. This country is an interesting case study since it exhibits a disproportionate share of workers in the public sector. Public sector employment is close to 22% of total employment (18% for men and 42% for women), about 7 percentage points higher than the average of 108 non-MENA economies.[3] Many of these employees are highly educated: among workers with college education, around 25% of men and two-thirds of women work in the public sector. Even though the model structure is simple, we show that it is capable to match several features of the allocation of talent in the Egyptian economy. Moreover, the tractability of the model allows to easily extend the analysis to several other developing economies.

We compute aggregate productivity gains, in terms of output per worker and total factor productivity (TFP), from adjusting the current public sector premium and the size of the public sector to their optimal levels. We show that reducing the public sector premium from the current to the optimal level would yield aggregate gains of 12% for output per worker and 8% for TFP if Egypt exhibits the elasticity of the private sector to public goods of the average developed economy (0.1, Bom and Ligthart 2014). The lower the value of this elasticity, the higher the productivity gains in our baseline economy from reducing the current public sector premium and the current share of public sector employment.

In addition, we show that, in Egypt, an optimal talent allocation (assuming an elasticity of private sector output to public goods of $\gamma = 0.1$) requires a decrease in the average public sector premium from 22% to 13%, but the premium would remain positive for women and close to zero for men. This wage structure reduces the size of the public sector from 22% to about 8%, and fosters female entrepreneurial activities. Moreover, compared to a wage

[3]MENA: Middle East and North Africa. See International Labor Organization (ILO) and https://theforum.erf.org.eg/2020/08/31/public-sector-employment-mena-comparison-world-indicators/

structure where the premium is equalized across genders, it prevents a significant transition of mid-high skilled women from the public sector to the home producer sector, and also, a contraction of the male entrepreneurial sector. Furthermore, we show that a lower female public sector premium (and a lower share of public sector employment) promotes female labor force participation in market activities. The latter stems from the expansion of the female entrepreneurial sector, which increases the demand for production labor and drives wages up. Consequently, low-skilled women are incentivized to transition from the home producer sector to the production labor force. Then, the number of low-skilled home producers contracts, more than compensating for the slight increase in the number of mid-high skilled women leaving the public sector and starting home production as a result of the lower public sector premium. This result then reverses the direction of causality between the demand for talent in the public sector and female labor force participation typically offered in the literature (Assad et al. 2020).

The literature frequently associates the existence of a public-private wage premium with an overpaid public sector (e.g., Calvalcanti and Rodrigues do Santos, 2021). Once such evidence is documented, these studies perform a counterfactual analysis that involves an arbitrary decrease of the public-private wage premium from its benchmark value. Moreover these frameworks discard, by design, the home-production sector. Our paper differs from that type of analysis in three dimensions. First, our model allows us to derive an optimal public sector premium. In other words, we show that the existence of a public-private wage premium does not necessarily reflect an overpaid public sector. Second, our counterfactual analysis is not arbitrary; we compute the efficiency gains of bringing the *current* public sector premium to its *optimal* level. Third, our economy model includes home production carried out by women. Thus, fewer incentives for women to join the public sector might result in their transition to the private sector or the home production sector. Each of these alternatives implies a different pool of talent involved in market activities and, thus, brings different consequences in terms of efficiency levels.

The rest of this paper is organized as follows. Section 2 develops our framework. Section 3 shows the comparative statics of our model. We present our calibration strategy in Section 4. Section 5 documents the results from some counterfactual experiments. Sections 6 presents the optimal public sector premium. We offer some concluding remarks in Section 7.

2 Model

Consider an economy populated by women and men, each endowed with s units of managerial skill (or talent) and one unit of time (inelastically supplied). The (given) skill distribution $G(s)$ is the same for women and men, with density $g(s)$ and support in the interval $S = [\underline{s}, \overline{s}]$. There is a mass one of economically active individuals who produce goods either in the market or at home. Women and men account for fractions ϕ_f and ϕ_m of this population, respectively, with $\phi_f + \phi_m = 1$.

There are three sectors in this economy: the private sector, the public or government sector, and the home production sector. Agents choose whether to work in the private or public sector. Women in addition have the option of using their talent and time to produce at home. In the private sector, individuals can run their own firm or become production workers and earn a wage; in the government, individuals work for a wage either as production workers or managing others. These managerial positions in the public sector receive a gender-specific premium, which we take as given.

2.1 Private sector

In the private sector there is a continuum of firms or businesses operated by entrepreneurs with heterogeneous managerial talent. Each production unit produces a homogeneous good combining labor l, capital k, and managerial skill s according to the following technology:

$$y = s^{1-\eta_e} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta_e}, \tag{1}$$

where $\eta_e \in (0,1)$ is the span-of-control parameter and $\alpha \in (0,1)$ denotes the share of capital in the variable factors.[4] As in Gonzalez and Parro (2022), the labor of women l^f and men l^m are combined using a CES aggregator with an elasticity of substitution given by σ:

$$l = \left[\left(l^f\right)^{\frac{\sigma-1}{\sigma}} + \left(l^m\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

[4] Our equilibrium concept involves a steady state and not a balanced growth path. Hence, we normalize aggregate productivity levels to one, which applies to the private, public, and home-production sectors.

Entrepreneurs maximize profits, taking wages and the rental price for capital services as given. The maximization problem of a type-i entrepreneur, for $i \in \{f, m\}$, with talent s is:

$$\max_{k_i, l_i^f, l_i^m} \pi_i(s) = y_i(s) - w_f l_i^f - w_m l_i^m - R k_i, \tag{3}$$

where R is the rental price for capital services, l_i^f the demand for female production labor by a type-i entrepreneur, l_i^m the demand for male production labor, k_i the demand for capital, and w_i is the wage of labor type-i.

The first-order conditions of the maximization problem result in the following system (see the Appendix):

$$y_i(s) = \Phi_e s, \tag{4}$$

$$k_i(s) = \left(\frac{\eta_e \alpha}{R}\right) y_i(s), \tag{5}$$

$$l_i(s) = \left(\frac{\eta_e(1-\alpha)}{\tilde{w}}\right) y_i(s), \tag{6}$$

$$l_i^f(s) = \left[1 + \left(\frac{w_f}{w_m}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} l_i(s), \tag{7}$$

$$l_i^m(s) = \left[1 + \left(\frac{w_m}{w_f}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} l_i(s), \tag{8}$$

where

$$\Phi_e = \left(\frac{\eta_e \alpha}{R}\right)^{\frac{\eta_e \alpha}{1-\eta_e}} \left(\frac{\eta_e(1-\alpha)}{\tilde{w}}\right)^{\frac{\eta_e(1-\alpha)}{1-\eta_e}},$$

$$\tilde{w} = \left[1 + \left(\frac{w_f}{w_m}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} w_f + \left[1 + \left(\frac{w_m}{w_f}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} w_m.$$

Note that $\eta_e < 1$ implies diminishing returns in variable inputs, which results in a non-degenerate endogenous size distribution of production units in the economy. Note also that in equilibrium entrepreneurial profits are linear in skill:

$$\pi_i(s) = (1-\eta_e) \Phi_e s \tag{9}$$

The total output produced in the entrepreneurial private sector is:

$$Y_e = G^{\gamma} y_E, \tag{10}$$

where y_E denotes the sum of production across entrepreneurial private activities, G corresponds to the public good produced by the government, and γ denotes the elasticity of aggregate private production, Y_e, to changes in the size of the public good G.[5]

2.2 Public sector

In the public sector there is a single public firm that produces a good G that increases the productivity in the private sector. The public firm combines managerial talent, production labor, and capital according to the following technology:

$$G(Z_p) = Z_p^{1-\eta_p} \left(K_p^{\kappa} L_p^{1-\kappa} \right)^{\eta_p}, \tag{11}$$

where Z_p is the aggregate amount of managerial talent attracted to the government, L_p denotes aggregate production labor, K_p is the aggregate capital, and $\eta_p \in (0,1)$ is the span-of-control parameter. Female production labor, L_p^f, and male production labor, L_p^m, are combined as in the private sector and with the same elasticity of substitution σ:

$$L_p = \left[(L_p^f)^{\frac{\sigma-1}{\sigma}} + (L_p^m)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{12}$$

To attract managerial talent into the public firm, the government pays a gender-specific premium, τ_f and τ_m, on the competitive wages of production workers w_f and w_m. Notice that this premium is not offered to wage workers in the public firm. In other words, while production workers earn the same rate in the public and private sector, type-i managers in the government earn a premium τ_i in excess of the wage rate. Definition 1 formally states what constitutes a managerial position within the public firm.

[5]The only role of the public good in this economy is to complement private sector production. That is, we abstain from potential welfare implications and assume that the good is not consumed or valued by workers. Given that the objective of the paper is to understand the effects of public-private talent allocation on aggregate productivity, this is a reasonable assumption.

Definition 1. *If an agent of type-i for $i \in \{f, m\}$ is hired by the government and earns a wage $(1+\tau_i)\, w_i$, then this agent is in a managerial position in the public firm.*

In the maximization problem of the public firm, we assume the government takes the premiums τ_f and τ_m as given. Let $I_i^p(s;\tau_i) = 1$ be an indicator for a type-i agent with skill s who, given τ_i, is hired by the public firm. Then the supply of managerial talent to the public sector, and therefore the size of the public good, will depend on the premium on managerial positions offered by the government:

$$Z_p = \phi_f \int_{s\in S} I_f^p(s;\tau_f)\, sg(s)\, ds + \phi_m \int_{s\in S} I_m^p(s;\tau_m) sg(s)\, ds, \tag{13}$$

Taking as given τ_f and τ_m, the maximization problem of the public firm is:

$$\max_{K_p, L_p^f, L_p^m} B = G(Z_p) - (1+\tau_f)\, w_f \tilde{L}_p^f - (1+\tau_m)\, w_m \tilde{L}_p^m - w_f L_p^f - w_m L_p^m - RK_p, \tag{14}$$

where

$$\tilde{L}_p^f = \phi_f \int_{s\in S} I_f^p(s;\tau_f)\, g(s)\, ds,$$
$$\tilde{L}_p^m = \phi_m \int_{s\in S} I_m^p(s;\tau_m)\, g(s)\, ds,$$

are the mass of female and male managerial talent hired by the public firm. These individuals take a managerial position in the government and earn a wage $(1+\tau_i)\, w_i$, while the rest of public workers are employed as production labor and earn w_i.

The first order conditions of the maximization problem result in the following public demand for inputs and size of the public good (see the Appendix):

$$G\left(Z_p\right) = \Phi_p Z_p, \tag{15}$$

$$K_p\left(Z_p\right) = \left(\frac{\eta_g \kappa}{R}\right) G\left(Z_p\right), \tag{16}$$

$$L_p\left(Z_p\right) = \left(\frac{\eta_g (1-\kappa)}{\tilde{w}}\right) G\left(Z_p\right), \tag{17}$$

$$L_p^f\left(Z_p\right) = \left[1 + \left(\frac{w_f}{w_m}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} L_p\left(Z_p\right), \tag{18}$$

$$L_p^m\left(Z_p\right) = \left[1 + \left(\frac{w_m}{w_f}\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} L_p\left(Z_p\right), \tag{19}$$

where

$$\Phi_p = \left(\frac{\eta_g \kappa}{R}\right)^{\frac{\eta_g \kappa}{1-\eta_g}} \left(\frac{\eta_g \left(1-\kappa\right)}{\tilde{w}}\right)^{\frac{\eta_g (1-\kappa)}{1-\psi}}.$$

An alternative setup of the public sector problem could have the premiums τ_f and τ_m determined exogenously together with the factors Z_p, K_p, L_p^f, and L_p^m. We believe that our setup is a better alternative that offers a lower bound on the productivity losses from a large public sector since the government is optimally setting the quantity and combination of inputs, and therefore the amount of the public good, given the public sector premiums. Disciplining the public sector with the government budget constraint, for example, would introduce unnecessary complications to our model. Indeed, the optimal size of the public sector is also a function of the distortionary taxes needed to fund it. Since this problem has already been dealt with in the literature, we choose to abstain from this margin to focus solely on the allocation of talent.[6]

2.3 Home production

Only women work in the home production sector. In other words, the home production sector consists of all activities performed by women who do not participate in the labor market and, thus, choose to produce at home. Home production requires specific skills and female labor, as

[6] As it is standard in the macro-development literature on productivity losses from different regulations, the implicit assumption is that the government is funding the public firm with non-distortionary lump-sum taxes.

outlined by the following technology:

$$h(s) = s^{1-\eta_h} \left(l_h^f\right)^{\eta_h} + \overline{h}, \tag{20}$$

where $\overline{h}$ is a level of household production that any woman can produce at zero cost. Consequently, more talented or skilled women will produce more of the home good compared to less talented women.

The maximization problem of a (female) home producer with talent s is

$$\max_{l_h^f} \tilde{\pi}_f(s) = h(s) - w_f l_h^f, \tag{21}$$

We refer to $\tilde{\pi}_f(s)$ as the (net) *value of home-produced goods* or *value of home production* for a woman with skill s. The first-order conditions of the maximization problem result in the following home output and home demand for inputs:

$$h(s) = \Phi_h s + \overline{h} \tag{22}$$

$$l_h^f(s) = \left(\frac{\eta_h}{w_f}\right)^{\frac{1}{1-\eta_h}} s, \tag{23}$$

where

$$\Phi_h = \left(\frac{\eta_h}{w_f}\right)^{\frac{\eta_h}{1-\eta_h}}.$$

Moreover, the value of home-production (for women only) is a linear function of skill:

$$\tilde{\pi}_f(s) = (1 - \eta_h)\Phi_h s + \overline{h} \tag{24}$$

Total output in the home production sector Y_h results from aggregating the activities of women who produce at home.

2.4 Occupational choices

To solve for the sorting of talent across both occupations and sectors we need to assume an allocation rule between managerial positions and wage-working positions in the public firm. Assumption 1 states this hiring rule:

Assumption 1. *The hiring rule of managerial talent in the public firm is such that the government accepts to hire an agent i with skills s for a managerial position if, and only if, $\pi_i(s) \geq w_i$.*

The allocation rule in Assumption 1 states that the government can observe the skill of job applicants and selects managerial talent by comparing the profits that an agent would generate as an entrepreneur to the production labor wage. If the former exceeds the latter, the government accepts the agent's job application for a managerial position in the public firm and pays a premium in excess of the production wage rate. Managerial positions in the public firm—those that earn the public sector premium—are then exclusively filled with workers with the highest managerial talent. Some countries have stringent regulations on the type of position public sector workers can occupy in terms of formal education degrees and experience, which is in line with the allocation rule in Assumption 1 (see for example the *Estatuto Administrativo* for Chile) [7]. Moreover, assuming that the most talented public sector employees become public sector managers while the less talented become public sector production workers is a relatively conservative scenario that offers a lower bound for the productivity losses from a large public sector. Alternative allocation rules inside the public firm (for example, if the less skilled individuals were to become public sector managers) would result in lower amounts of the public good.

Agents select the occupation that maximizes their earnings. Men compare their profits as entrepreneurs $\pi_m(s)$, their earnings as production workers w_m (whether in the public or the private sector), and their earnings as managers in the government $(1 + \tau_m) w_m$ (although these positions are only available for the most skilled men). In equilibrium, a male entrepreneur earns profits greater than the wage offered by the government for managerial talent. Let $I_i^e(s; \tau_i)$ be

[7] https://www.dt.gob.cl/portal/1626/articles-117137_galeria_36.pdf

an indicator for a type-i agent with skills s who optimally becomes an entrepreneur. Then,

$$I_m^e(s;\tau_m) = \begin{cases} 1 \text{ if } \pi_m(s) \geq (1+\tau_m)\, w_m \text{ and } \pi_m(s) \geq w_m \\ 0 \text{ otherwise.} \end{cases} \tag{25}$$

Notice that a man for whom $I_m^e(s;\tau_m) = 1$ would be eligible for a managerial position in the public firm, but the premium paid by the government is not sufficiently high to reward his (relatively high) talent. In this case, the agent would choose not to become a manager in the public firm and would instead prefer to use their managerial talent in the private sector.

The premium offered by the government would attract some men who possess enough talent to become entrepreneurs in the private sector. As entrepreneurs, these individuals would be capable of generating profits exceeding the production labor wage but less than the wage paid by the government, including the premium. Moreover, Assumption 1 implies that the public sector would hire these agents to fill managerial positions. Consequently, we have:

$$I_m^p(s;\tau_m) = \begin{cases} 1 \text{ if } \pi_m(s) \geq w_m \text{ and } \pi_m(s) < (1+\tau_m)\, w_m \\ 0 \text{ otherwise.} \end{cases} \tag{26}$$

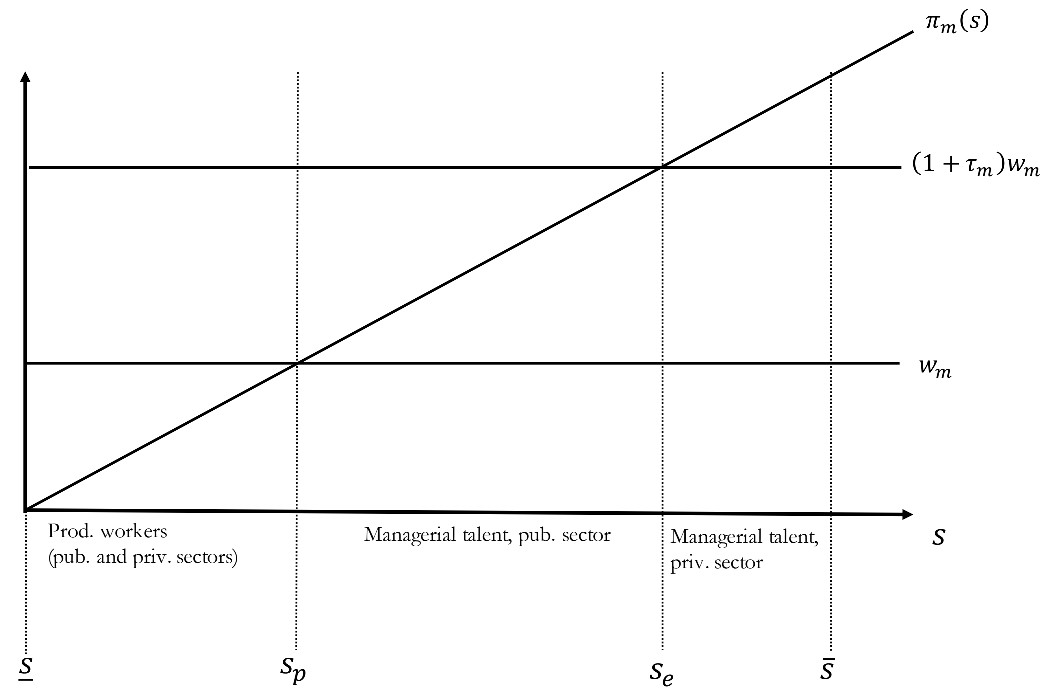
Men who neither become entrepreneurs nor supply their managerial talent to the public firm join the production labor workforce. Let $I_i^w(s;\tau_i)$ be an indicator for an agent who optimally becomes a production worker. Then,

$$I_m^w(s;\tau_m) = 1 - I_m^e(s;\tau_m) - I_m^p(s;\tau_m) \tag{27}$$

Figure 1 illustrates an equilibrium allocation of male talent across occupations. In equilibrium, there are two cutoffs s_p and s_e in the male skill distribution such that $I_m^e(s;\tau^m) = 1$ for $s \in [s_e, \overline{s}]$; $I_m^p(s;\tau_m) = 1$ for $s \in [s_p, s_e)$; and $I_m^w(s;\tau_m) = 1$ for $s \in [\underline{s}, s_p)$. Men with skills $s \in [s_e, \overline{s}]$ are talented managers who earn high profits in the private sector. As a result, the premium τ_m is insufficient to attract this managerial talent to the public sector and these individuals work in the private sector running their own businesses.

Individuals with skills $s \in [s_p, s_e)$ would become entrepreneurs without the public sector premium. However, the public firm successfully attracts them into public managerial positions over private business ownership (and they are hired according to Assumption 1). The least talented individuals, those with skills $s \in [\underline{s}, s_p)$, do not have enough managerial skill to run private businesses or to be hired by the public firm in managerial roles. They become production workers and are employed either in a private business or in the public firm.

Figure 1: Allocation of male talent



Women compare their value of home production $\tilde{\pi}_f(s)$, their profits as entrepreneurs $\pi_f(s)$, their earnings as production workers w_f (whether in the public, the private sector, or the home production sector), and their earnings as managers in the government $(1+\tau_f)w_f$ (although these positions are only available for the most skilled women).

Women participate in the labor market only when their potential earnings exceed the net value of home-produced goods. More specifically, women become entrepreneurs when they would operate businesses that generate profits higher than the wage paid by the public firm for managerial talent and also higher than the net value of the output they can produce at home:

$$I_f^e(s;\tau_f) = \begin{cases} 1 \text{ if } \pi_f(s) \geq (1+\tau_f)w_f \text{ and } \pi_f(s) \geq w_f \text{ and } \pi_f(s) \geq \tilde{\pi}_f(s) \\ 0 \text{ otherwise.} \end{cases} \tag{28}$$

Like their male counterparts, some potential female entrepreneurs hold instead managerial positions in the public firm. These women receive a premium that compensates them enough for both their potential entrepreneurial profits and the value of goods they could produce at home:

$$I_f^p(s;\tau_f) = \begin{cases} 1 \text{ if } \pi_f(s) \geq w_f \text{ and } \pi_f(s) < (1+\tau_f)\,w_f \text{ and } \tilde{\pi}_f(s) \leq (1+\tau_f)\,w_f \\ 0 \text{ otherwise.} \end{cases} \tag{29}$$

Define $I_f^{wh}(s;\tau_f) = 1 - I_f^e(s;\tau_f) - I_f^p(s;\tau_f)$. Then $I_f^{wh}(s;\tau_f) = 1$ for a woman who does not supply their managerial talent, either in the private or the public sector; she either becomes a production worker or produces at home. In addition, let $H(s;\tau_f) = 1$ for a woman whose net value of home production exceeds the production labor wage. That is $H(s;\tau_f) = 1$ if $\tilde{\pi}_f(s) \geq w_f$ and $H(s;\tau_f) = 0$ if $\tilde{\pi}_f(s) < w_f$. Finally, let $H_f(s;\tau_f)$ be an indicator for a woman who becomes a home producer. Then,

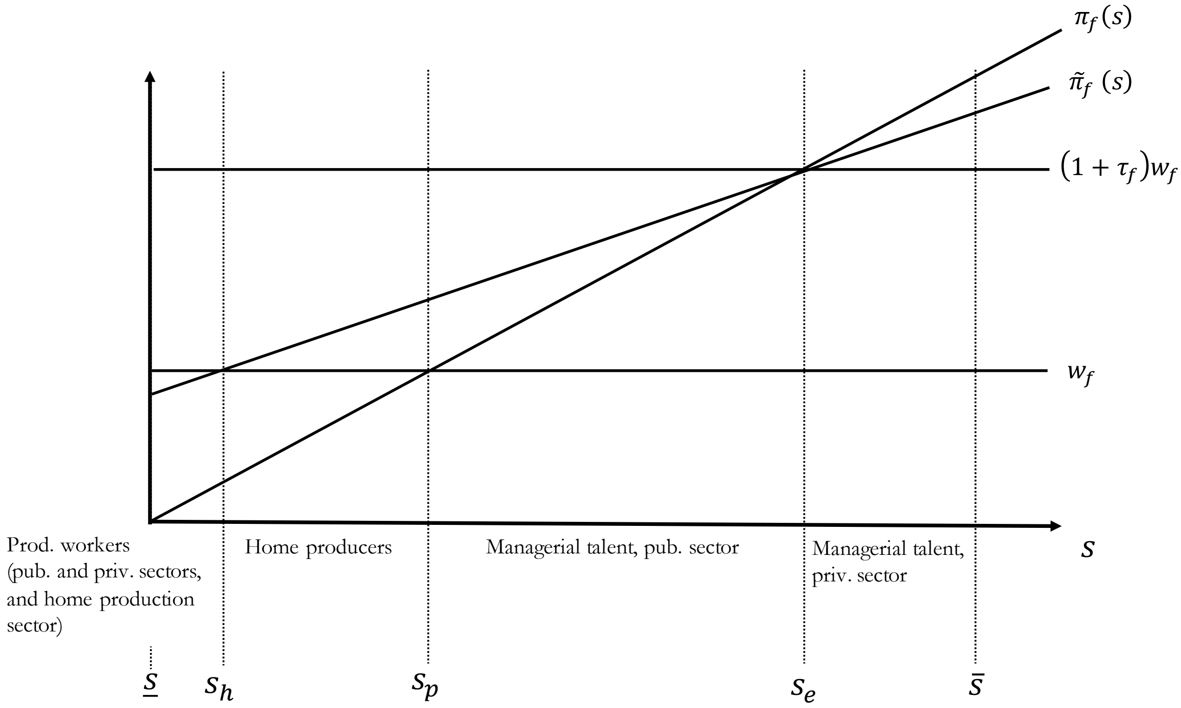
$$H_f(s;\tau_f) = \begin{cases} 1 \text{ if } I_f^{wh}(s;\tau_f) * H(s;\tau_f) = 1 \\ 0 \text{ otherwise.} \end{cases} \tag{30}$$

Women who optimally becomes a production worker are then:

$$I_f^w(s;\tau_f) = 1 - I_f^e(s;\tau_f) - I_f^p(s;\tau_f) - H_f(s;\tau_f) \tag{31}$$

Figure 2 illustrates an equilibrium allocation of talent for a case in which $(1-\eta_e)\,\Phi_e > (1-\eta_h)\,\Phi_h$ and $\overline{h} < w_f$. The latter implies that the dispersion of profits is higher than the dispersion in the value of home production, while the former means that the value of free home-made output $\overline{h}$ is smaller than the wage earned by production labor workers. Under these conditions, there are three cutoffs s_h, s_p, and s_e in the female skill distribution such that $I_f^e(s;\tau_f) = 1$ for $s \in [s_e, \overline{s}]$ (these women become entrepreneurs); $I_f^p(s;\tau_f) = 1$ for $s \in [s_p, s_e)$ (these women become managers in the public firm); $H_f(s;\tau^f) = 1$ for $s \in (s_h, s_p)$ (these women do not enter the labor force); and $I_f^w(s;\tau_f) = 1$ for $s \in [\underline{s}, s_h]$ (these women become production workers).

Figure 2: Allocation of female talent



In the equilibrium illustrated in Figure 2, only the most talented women run businesses in the private sector. As with their male counterparts, the female premium set by the public firm is not high enough to attract them into the public sector. Additionally, these women are not able to produce a value at home as high as the profits they earn as entrepreneurs in the private sector.

The public firm identifies women with skills $s \in [s_p, s_e)$ as women with managerial talent because, as potential entrepreneurs, they are capable of generating profits greater than the production labor wage. Consequently, the public firm offers a managerial position and they accept because their opportunity cost as entrepreneurs is less than their earnings in the public sector (including the premium).

Women with skills $s \in [s_h, s_p)$ do not participate in the labor market and, instead, engage in home production. These women are not hired by the government for managerial positions since their managerial talent is not high enough (they would not generate profits above the production labor wage and by Assumption 1, the public firm does not recognize them as talented and is not willing to pay them the premium). These women also do not become production workers because the value of their home production exceeds the wage. The least talented women become production workers. These women are not very productive at home, nor are they productive as entrepreneurs. They also lack the necessary talent to be hired by the public firm for managerial positions.[8]

2.5 Market clearing

Assumption 1 directly implies that the market clearing condition for managerial talent in the public sector is:

$$\phi_f \int_{s \in S} I_f^p (s; \tau_i) \, g(s) \, ds = \tilde{L}_p^f, \tag{32}$$

$$\phi_m \int_{s \in S} I_m^p (s; \tau_i) \, g(s) \, ds = \tilde{L}_p^m. \tag{33}$$

Male production workers are employed either in a private business or in the public firm. In the case of women, they can also be employed by other women who produce at home. Then, the labor market conditions for female and male production labor are:

$$\phi_f \int_{s \in S} I_f^w (s; \tau_f) g(s) \, ds = \phi_f \int_{s \in S} I_f^e (s; \tau_f) l_f^f (s) \, g(s) \, ds + \phi_m \int_{s \in S} I_m^e (s; \tau_m) l_m^f (s) \, g(s) \, ds + \phi_f \int_{s \in S} H_f (s; \tau_f) l_h^f (s) \, g(s) \, ds + L_p^f, \tag{34}$$

$$\phi_m \int_{s \in S} I_m^w (s; \tau_f) g(s) \, ds = \phi_f \int_{s \in S} I_f^e (s; \tau_f) l_m^f (s) \, g(s) \, ds + \phi_m \int_{s \in S} I_m^e (s; \tau_m) l_m^m (s) \, g(s) \, ds + L_p^m. \tag{35}$$

Lastly, we assume an economy that faces an infinitely elastic supply of capital consistent with a interest rate $R = 1/\beta + \delta - 1$. This can be directly derived from the Euler Equation that the optimization problem of a representative household must satisfy in the steady state.[9] Then,

[8] Notice that the model predicts that, in the public sector, less skilled managers are more "overpaid" than those with more skills. Specifically, consider the profits an individual can earn as an entrepreneur in the private sector as a proxy for her talent. As observed in Figures 1 and 2, the gap between the total wage earned by public sector managers and their potential profits as private sector entrepreneurs decreases with talent. This prediction is consistent, for example, with the evidence documented by Cerda and Pessino (2018).

[9] The Appendix provides further details.

the aggregate stock of capital is determined by the demand side of the economy:

$$K = \phi_f \int_{s \in S} k_f(s) I_f^e(s; \tau_f) g(s) ds + \phi_m \int_{s \in S} k_m(s) I_m^e(s; \tau_m) g(s) ds + K_p. \tag{36}$$

2.6 Competitive equilibrium

The equilibrium of this economy is an allocation $\{K\}$, a set of occupational choices $\left\{I_f^e(s), I_f^p(s), I_f^w(s), H_f(s)\right\}_{s\in S}$ and $\{I_m^e(s), I_m^p(s), I_m^w(s)\}_{s\in S}$, and a set of prices $\{R, w_f, w_m\}$ such that, given the premiums τ_f and τ_m, (i) agents choose their occupation to maximize earnings, (ii) entrepreneurs choose the amount of capital and labor to maximize profits, (iii) the government maximizes its surplus (see (14) above), and (iv) labor and capital markets clear.

3 Comparative statics

In this section we show some of the qualitative properties of the model in a partial equilibrium context. Specifically, we explore the allocation of female and male talent across the three sectors of the economy when the managerial premium in the public sector falls.

Figure 3 depicts a comparative statics exercise in which the male premium in the public sector decreases. This reduction in the managerial public sector premium leads to a reallocation of managerial talent from the public sector into the private sector (keeping prices constant). As a result, the size of the public firm falls and the size of the private sector increases. Importantly, the private sector now operates with a smaller quantity of the public good G. Therefore, production in the private sector is carried out by more firms, but the production in these firms is complemented by a smaller amount of the public good.

In contrast, an increase in the premium paid to managers in the public sector would reduce private sector employment, leading to the existence of fewer (but bigger) businesses. These firms, however, would operate with a larger amount of the public good that complements their productive activities since the public firm recruits more managerial talent, resulting in a greater span of control over variable inputs. Combined, these results suggest the existence of an optimal (managerial) premium in the public sector. On the one hand, a very high premium would increase the size of the public good that private businesses need to produce but would leave the private sector without enough managerial talent. On the other hand, a very low premium would leave the public firm without enough managerial talent, thus lowering the production of the public good, but would increase the size of the private sector. The optimal level of the public sector premium is the one that balances this trade-off and results in an allocation of talent between the private and public sectors that maximizes some aggregate output. Importantly, the optimal premium also implies an optimal size of the government since, in this economy, it is the premium that determines the amount of talent attracted to the public sector, and thus, the size of the public firm. Any premium that deviates from the optimal level will therefore distort the allocation of talent in the economy, undermining aggregate output and productivity.

Figure 3: Allocation of male talent, fall in the premium

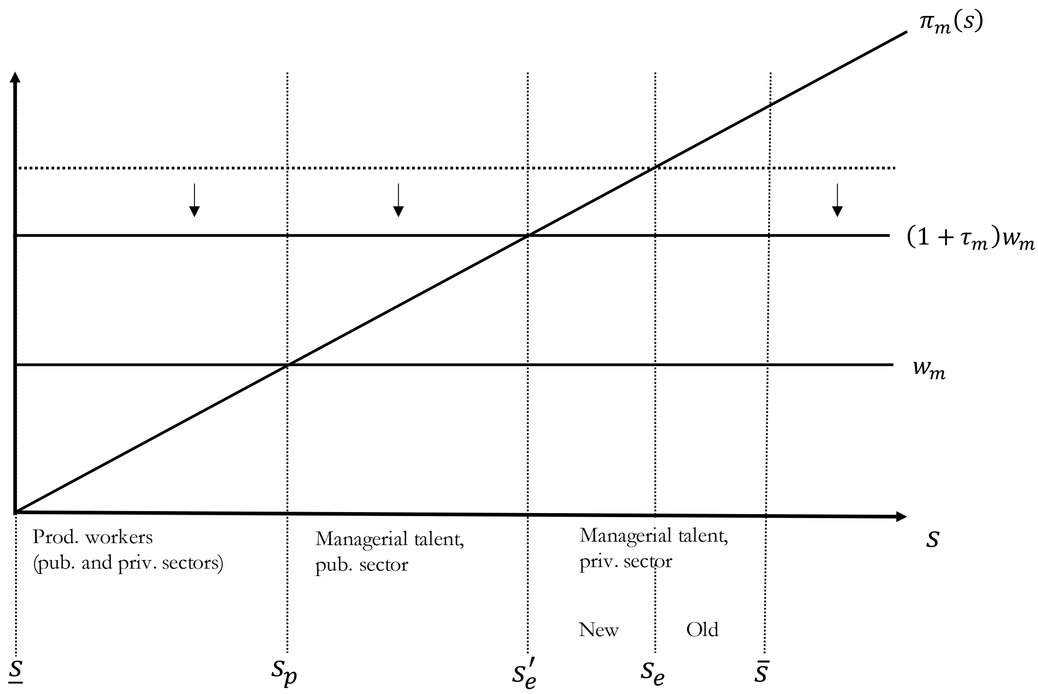
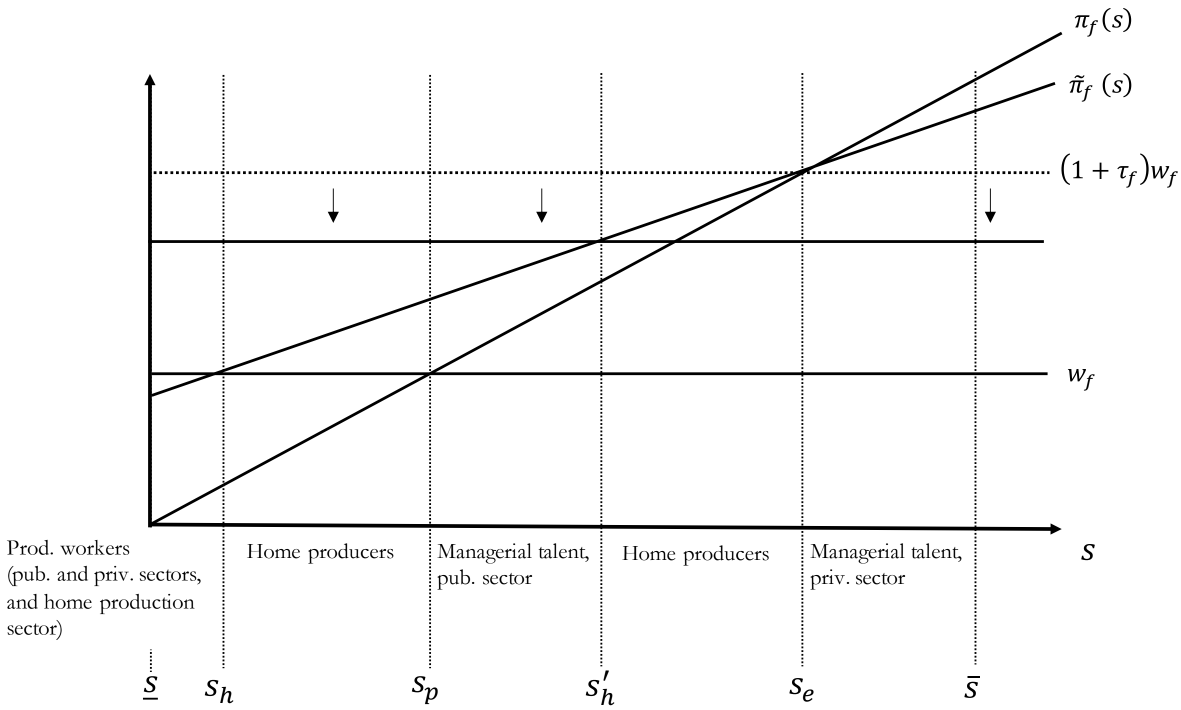


Figure 4 shows the effects of a reduction in the (managerial) public sector premium on the female allocation of talent. As is the case of male workers, a decrease in the premium results in a reallocation of managerial talent away from public firms. Interestingly, women who leave managerial positions in the public firm do not necessarily transition into another occupation in the labor market but in this example they instead become home producers.

Then, a fall in the female public sector premium triggers two main partial equilibrium effects. First, the amount of the public good decreases as a consequence of the public firm hiring less managerial talent. Then, even though the mass of female entrepreneurs in the private firms in this example remains the same, they produce with a smaller amount of the public input. Second, home production increases. The optimal level for the public sector premium depends therefore not only on the allocation of female talent in the labor market but also on whether home production is considered in the analysis. The empirical section of this paper will revisit this issue in more detail.

Figure 4: Allocation of female talent, fall in the premium



Overall, this partial equilibrium analysis offers several insights on the allocation of talent between the public sector, the private sector, and home production. First, a high public sector premium results in a high employment share in the government among the relatively talented agents. Second, and related to the previous point, a high public premium expands the size of the public sector and shrinks the private sector. Third, changes in the public premium could reallocate women either out of home production into the public sector or from the public sector into home production, that is, in or out of the labor market. Fourth, there exists an optimal public sector premium, and any deviation from that level distorts the allocation of talent in the economy, leading to aggregate consequences on output and productivity. Fifth, the inclusion of the home production sector is indeed relevant for assessing the efficiency gains of a rise or fall in the premium paid in the public sector.

4 Calibration strategy

We calibrate the parameters in our model to match key moments of the allocation of talent in Egypt. Our data were extracted from ILOSTAT databases.[10]

We select α such that $\eta_e\alpha$ (the share of capital) equals a capital income share of 30%, as is standard in the literature (e.g., Gollin, 2002). We assume that the capital income share in the public firm is the same as in the private sector. Hence, we determine κ such that $\eta_g\kappa$ equals 0.3. The fractions of women and men in the population, ϕ_f and ϕ_m, are directly taken from the data: $\phi_f = \phi_m = 0.5$. For the parameters of the Euler equation used to set the location of the infinitely elastic supply of capital (β and δ), we take the standard value of 0.95 for the discount factor β, as reported in the literature, and we follow Cagetti and De Nardi (2006) and Buera and Shin (2013) to choose $\delta = 0.06$.[11]

In line with Guner et al. (2008), we assume that log-managerial ability follows a (truncated) normal distribution with a mean of m and a standard deviation of sd. Additionally, we impose that this distribution represents the majority of production units, constituting a total mass of $1 - \hat{p}$. To address the remaining portion of the establishment distribution, we set a top value for managerial skill, $\hat{s}$, along with its corresponding fraction, $\hat{p}$. We normalize $\hat{p} = 0.1\%$ and let $\hat{s}$ to match the employment share in firms with 20 or more employees. Note that m, sd, and the span of control parameter, η_e, directly determine the distribution of firms and the composition of female and male entrepreneurs. Consequently, we calibrate these parameters to match the employment share in firms with 20 or more employees, firms with 1-9 employees, the proportion of female entrepreneurs, and the proportion of male entrepreneurs.

[10] International Labour Organization (ILO), accessed on April 2024. The share of employment in the public sector in the data includes workers in State-Owned Enterprises.

[11] This value for the depreciation rate is common in the business cycle literature (Cooley, 1995; Cagetti and De Nardi, 2006; Heathcote et al., 2010). Furthermore, Stokey and Rebelo (1995) show that calculations based on the capital consumption allowance and estimates of the aggregate capital stock yield an average depreciation rate of around 6%.

The elasticity of substitution between women and men, denoted as σ, is calibrated to replicate the observed gender wage gap in the benchmark economy. Similarly, we set the span of control parameter for the public firm, η_g, to match the fraction of public employment relative to total employment. The home production technology is characterized by two parameters: η_h and $\overline{h}$. We adjust these parameters to match the fraction of female production workers relative to total female employment and a metric indicating the proportion of women engaged in home production. While the former variable can be directly obtained from data, the latter requires proxying. To address this, we first calculate the number of women aged 15-64 who are not part of the labor force. Then, we assume that one-third of this group engages in home production and determine the fraction they represent relative to the combined total of the female labor force and those involved in home production. In other words, we do not assume that women not in the labor force are automatically engaged in home production since women who are neither in the labor force nor engaged in home producing could be enrolled in educational institutions or be dependent on other household members, for example. Under our assumption, we find that 60% of working-age women are engaged in home production. Notice that this figure is consistent with the reported difference in the participation in unpaid care work between women and men in Egypt. Furthermore, it is also consistent with the fraction of weekly time that women spend on home production activities (as reported) in Egypt.[12] In addition, Nazier and Ezzat (2018) also show that, in Egypt, women allocate around 60% of their time to housework.

We calibrate the public sector premium, τ_f and τ_m, to match the fraction of female and male public employment. These parameters can also be used to compute the ratio of the average wage in the public sector over the production labor wage (minus one), which we denote by $\tilde{\tau}_f$ and $\tilde{\tau}_m$ for women and men, respectively. To distinguish between the two, we continue to refer to τ_i as the *premium* and refer to $\tilde{\tau}_i$ as the *average premium*.

Finally, note that the parameter γ, which represents the elasticity of aggregate private production to the public good G, is not required to calibrate the model since our targets do not involve aggregate output. The calibrated talent allocation is therefore consistent with different levels of γ. However, as we will show later, different levels of public sector efficiency,

[12] See https://arabstates.unwomen.org/sites/default/files/Field%20Office%20Arab%20States/Attachments/Publications/2020/12/UNW_ERF_PolicyBriefs_Egypt_final_Dec4_2020.pdf

γ, imply different optimal premiums, which in turn impact the optimal talent allocation in the economy. In our exercises, we treat γ as a free parameter that represents the level of public sector efficiency, and compute the optimal premium for a given level of γ.

Table 1 lists the model parameters and their respective sources, while Table 2 summarizes the performance of the model in terms of the targeted moments.[13] Notice that in our baseline economy, the calibrated public sector premium averages 37% for women but less than 1% for men. The calibrated average across men and women is 22%.

Table 1: Parameters of the model

Share of capital, private sector ($\eta_e \alpha$)	0.3	Literature, e.g., Gollin, 2002
Share of capital, public firm ($\eta_g \kappa$)	0.3	Literature, e.g., Gollin, 2002
Fraction of women (ϕ_m)	0.5	Data (exogenous)
Fraction of women (ϕ_m)	0.5	Data (exogenous)
Discount factor (β)	0.95	Literature
Depreciation rate (δ)	0.06	Literature, e.g., Cooley, 1995; Stokey and Rebelo, 1995; Heathcote et al., 2010; Buera and Shin, 2013 Cagetti and De Nardi, 2006
CES elasticity (σ)	8.6	Calibrated to match data
Mean log-entrepreneurship skills (m)	–0.4	Calibrated to match data
Variance of log-entrepreneurship skills (sd)	18.5	Calibrated to match data
Highest entrepreneurship ability ($\hat{s}$)	185,000	Calibrated to match data
Mass of highest entrepreneurship ability ($\hat{p}$)	0.1%	Normalized
Span-of-control, private sector (η_e)	0.53	Calibrated to match data
Span-of-control, public sector (η_g)	0.86	Calibrated to match data
Home production parameter I ($\bar{h}$)	42	Calibrated to match data
Home production parameter II (η_h)	0.41	Calibrated to match data
Public sector premium, women (τ_f)	0.76	Calibrated to match data
Public sector premium, men (τ_m)	0.07	Calibrated to match data
Public sector premium, average, women ($\tilde{\tau}_f$)	0.37	Directly computed from the model parameters post-calibration
Public sector premium, average, men ($\tilde{\tau}_m$)	<0.01	Directly computed from the model parameters post-calibration

[13]The purpose of our paper is not to provide a theory to explain the calibrated gender wage gap. It likely encompasses a mix of the factors studied in the literature (e.g., Blau and Kahn, 2017).

Table 2: Performance of the model

Statistic	Data	Model
Gender wage premium	1.0	0.94
Fraction of female entrepreneurs	0.33	0.34
Fraction of male entrepreneurs	0.28	0.25
Employment share in firms with 1-9 employees	0.57	0.63
Employment share in firms with 20 or more employees	0.36	0.36
Fraction of public employment over total employment	0.22	0.25
Fraction of female production workers over total female employment	0.25	0.26
Fraction of women that produce at home	0.6	0.53
Fraction of public employment among women	0.42	0.41
Fraction of public employment among men	0.18	0.18

Notes: (i) The gender wage premium is the ratio of the average earnings of men to the average earnings of women. The model equivalent measure is the ratio of the production labor wage of men to the production labor wage of women. (ii) The fraction of female (male) entrepreneurs is the ratio of individuals who hold jobs where remuneration is directly dependent upon the profits derived from the goods and services produced to total female (male) employment. The model equivalent measure is the ratio of the mass of female (male) entrepreneurs to the mass of women (men) who are either production workers, public sector managers, or entrepreneurs. (iii) The employment share in firms with x employees is the ratio of workers in firms with x employees to total employment. (iv) The fraction of public employment over total employment is the ratio of the number of workers in the public sector to total employment. The model equivalent measure is the ratio of the summation of production labor in the public firm and public sector managers to the summation of production labor, public sector managers, and entrepreneurs. (v) The fraction of female production workers over total female employment is the ratio of female private employment to total female employment minus the ratio of female entrepreneurs to total female employment. The model equivalent measure is the ratio of the mass of total female production workers minus female production workers in the public firm to the summation of the mass of female production workers, female public sector managers, and female entrepreneurs. (vi) The share of women who produce at home is computed as a third of the number of working wage women minus the female labor force. The fraction of women who produce at home is the ratio of the number of women who produce at home to the summation of women who produce at home and the female labor force. The model equivalent measure is the ratio of home producers to the summation of female production workers, female public sector managers, female entrepreneurs, and home producers. (vii) The fraction of public employment among women (men) is the ratio of the number of female (male) workers in the public sector to total female (male) employment. The model equivalent measure is the ratio of the summation of female (male) production labor in the public firm and female (male) public sector managers to the summation of female (male) production labor, female (male) public sector managers, and female (male) entrepreneurs.

5 Efficiency gains from a reallocation of talent

One of the primary goals of this paper is to shed light on the consequences on allocative efficiency from different levels of the public sector premium (and therefore, different sizes of the public sector). The model accommodates several measures of aggregate productivity in the allocation of talent. These measures differ in the way labor and/or output are aggregated. Specifically, home production may or may not be accounted for in aggregate output, while labor could be aggregated in terms of raw units (production labor) or in terms of efficiency units. We focus on productivity measures that are truly observable for policymakers: output per worker and aggregate TFP, both based on market output and the aggregation of production labor.

We define output per worker, q, as:

$$q = \frac{Y_e}{L_1 + L_2}, \tag{37}$$

where Y_e is the total production carried out by both female and male entrepreneurs in the private sector:

$$Y_e = G^\gamma \left(\phi_f \int_{s \in S} y_f(s) \, I_f^e(s; \tau_f) g(s) \, ds + \phi_m \int_{s \in S} y_m(s) \, I_m^e(s; \tau_m) g(s) \, ds \right), \tag{38}$$

and L_1 and L_2 are the total amounts of production labor allocated to the private sector and the public sector, respectively:

$$L_1 = \phi_f \int_{s \in S} \left(l_f^f(s) + l_f^m(s) \right) I_f^e(s; \tau_f) g(s) \, ds + \phi_m \int_{s \in S} \left(l_m^f(s) + l_m^m(s) \right) I_m^e(s; \tau_m) g(s) \, ds. \tag{39}$$

$$L_2 = L_p^f + L_p^m. \tag{40}$$

and L_p^f and L_p^m are the expressions defined in Section 2.2.[14]

In addition, we define TFP as the residual from an aggregate technology under a capital share of $\alpha\eta_e$ and labor share of $1 - \alpha\eta_e$.[15] Then,

$$TFP = \frac{Y_e / (L_1 + L_2)}{(K / (L_1 + L_2))^{\alpha\eta_e}}. \tag{41}$$

Tables 3 and 4 below show how talent would reallocate across sectors and across occupations if the premium in our baseline economy would decrease from the calibrated level (averaging 22%) to the optimal (averaging 13%), assuming a level of public efficiency of $\gamma = 0.1$. In this economy, the productivity gains from a change in the public sector premium, and therefore the size of the public sector, depend critically on the size of γ, the elasticity that captures

[14]Home production is excluded from the numerator of q. Consequently, we do not include production labor employed at home in the labor aggregator implied by q. Suppose, in contrast, that production labor at home were included in the labor aggregator. Then, by construction, the output produced by these workers would not be accounted for, but they would be included in the accountable definition of the labor force. This would result in an unconvincing way of computing the efficiency level of the economy.

[15]We maintain consistency in how output and labor are aggregated; that is, home production is excluded from output, and home-employed production labor is excluded from the labor aggregator.

the responsiveness of aggregate production to changes in the public good. Bom and Ligthart (2014) conducted a systematic meta-regression analysis to quantify the private output elasticity of the stock of public capital, a parameter that in their setup corresponds to the elasticity γ in ours.[16] Bom and Ligthart look at a sample of 578 estimates from 68 studies and find a short run elasticity of 0.083, which increases to 0.122 in the long run. The average is 0.106. Given that their evidence is from developed economies only, we believe that a value of 0.1 is a conservative estimate (likely an upper bound) for γ in our baseline economy.

Among women, reducing the (average) public sector premium, and therefore the size of the public sector, results in more entrepreneurship, more production employees in the private sector, and less home production. Among men, the reduction in the premium and the size of the public sector results in more production workers in the private sector but the reallocation across occupations is negligible. A reduction in the public sector premium for women (to its optimal level) prompts mid-to-high skill women to move away from the public sector. A fraction of these women transition into entrepreneurship, while others start home production. The expansion of the female entrepreneurial sector drives up the demand for production labor, leading to an increase in wages. This rise in wage encourages the least skilled women, previously engaged in home production, to enter the production labor force. Consequently, the female entrepreneurial sector expands, the number of female production workers increases, and the pool of female managerial talent in the public sector shrinks. Moreover, the home production sector contracts but undergoes a shift in its composition: the proportion of mid-to-high skilled women relative to low-skilled women engaged in home production rises. Furthermore, with reduced demand from the public sector and the female home production sector, the composition of female production labor changes: labor allocated to the private sector increases, while that allocated to the public sector and the home production sectors decreases. In addition, the higher demand for production labor also pushes male wages up.[17] As male wages increase and the male public

[16] In our model, G corresponds to a flow of services from public goods. These goods increase private sector output Y_e with an elasticity of aggregate private production to the size of the public good γ. The parameter estimated in the meta-regression analysis of Bom and Ligthart (2014) is the data-equivalent of our model parameter, but for developed economies. Bom and Ligthart consider studies that estimate the output elasticity of public capital θ in a production function of the form $Y = AK^{\alpha}L^{\beta}G^{\theta}$. The idea behind this approach is that the services of public capital are proportional to the stock of public capital. In most of the studies they consider, public capital includes core infrastructure (roads, railways, airports, utilities), hospitals, educational buildings, and other public buildings. The output measure in many cases is real gross output of the private sector (excluding taxes and subsidies on products) or real GDP exclusive of public sector output.

[17] The rise in wages does not need to be large since the distribution of talent is such that the mass of low-skilled

sector slightly decreases, the size of the male entrepreneurial sector undergoes minimal change, and male production labor shifts from the public to the private sector. Reducing the public sector premium from the current to the optimal level yields aggregate efficiency gains in our baseline economy of 12% for q (output per worker) and 8% for our measure of TFP.

Table 3: Distribution of female talent

	Baseline	Optimal
Entrepreneurs	0.16	0.20
Managerial talent, pub. sector	0.09	0.04
Production workers	0.22	0.47
Private sector	0.50	0.89
Public sector	0.44	0.07
Home production sector	0.07	0.04
Home producers	0.52	0.29
Home producers, low skill	1	0.95
Home producers, high skill	0	0.05

Note: The baseline scenario corresponds to the calibrated premium. The counterfactual scenario corresponds to the optimal premium for $\gamma = 0.1$.

Table 4: Distribution of male talent

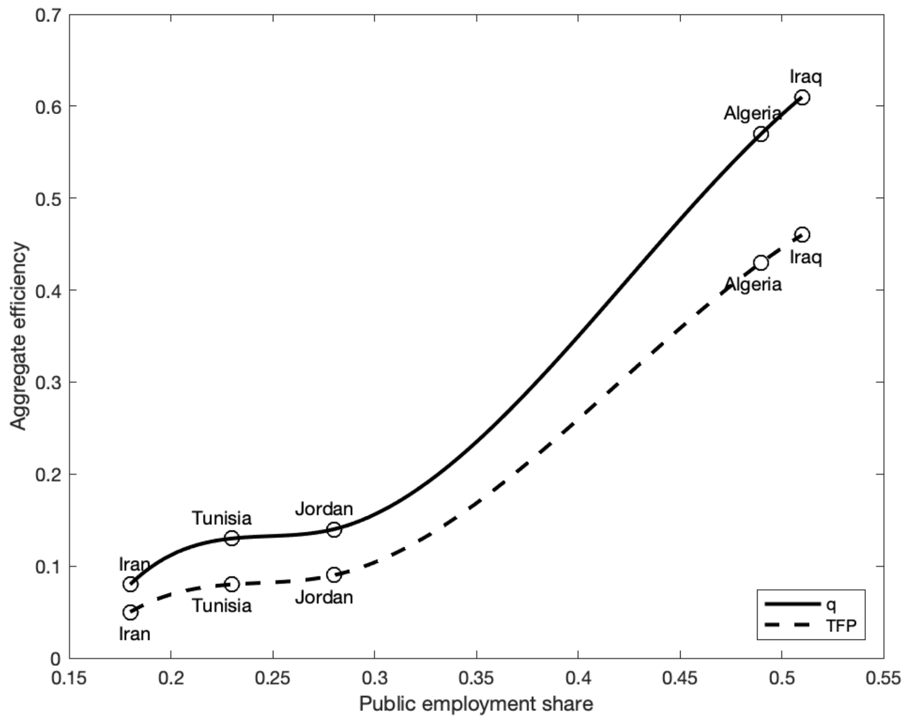
	Baseline	Optimal
Entrepreneurs	0.25	0.26
Managerial talent, pub. sector	0.01	<0.01
Production workers	0.74	0.74
Private sector	0.76	0.93
Public sector	0.24	0.07

Note: The baseline scenario corresponds to the calibrated premium. The counterfactual scenario corresponds to the optimal premium for $\gamma = 0.1$.

In Figure 5 we show the efficiency gains from a reallocation of talent to the optimal for different sizes of the public sector (measured using the share of public sector employment). The figure assumes a level of public sector efficiency γ of 0.1. As a reference, we show the levels of public sector employment (and potential gains from reallocation) in different MENA countries. The figure shows that for a given value of public sector efficiency, the potential efficiency losses from talent misallocation are larger the larger the share of employment in the public sector.

individuals is relatively sizable.

Figure 5: Efficiency gains from a reallocation of talent for different shares of public sector employment



Note: The counterfactual scenario corresponds to the allocation under the optimal premium for $\gamma = 0.1$. Circles correspond to values computed for each country, whereas the remaining points on the curve were interpolated. Shares in public sector from ILOSTAT databases.

6 The optimal public sector premium

We compute next the optimal *average* public sector premium that maximizes aggregate productivity (either output per worker q in indicator (40) or TFP in indicator (41) for different values of the elasticity γ. Recall that we define the average public sector premium as the ratio of average wage in the public sector to average wage in the private sector (minus one). The optimal level of the average public sector premium implies in turn an optimal size for the public sector.

The top panel in Figure 6 shows the optimal average public sector premium in our benchmark economy (Egypt) for different values of γ, which represents different levels of public sector efficiency.[18] We observe in the figure that the higher the elasticity of private sector output to

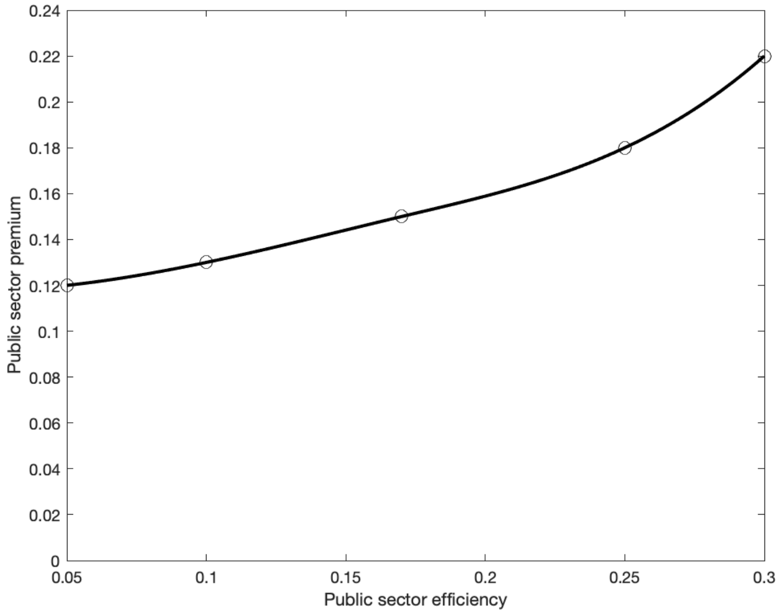
[18]Recall that γ is the elasticity of private sector output to the public sector good.

the public good, the higher the optimal premium. In other words, if the observed premium is high, an efficient allocation of talent would require a high elasticity of output to the public sector. At very low levels of public sector efficiency (i.e., $\gamma = 0.05$), the optimal premium reaches only 12%. Note that the baseline (calibrated) premium of 22% for Egypt is only optimal when γ is close to 0.3. For a value of $\gamma = 0.1$, as in the average developed economy as documented by Bom and Ligthart (2014), the calibrated premium in our baseline economy is significantly higher than the optimal (22% vs. 13%).

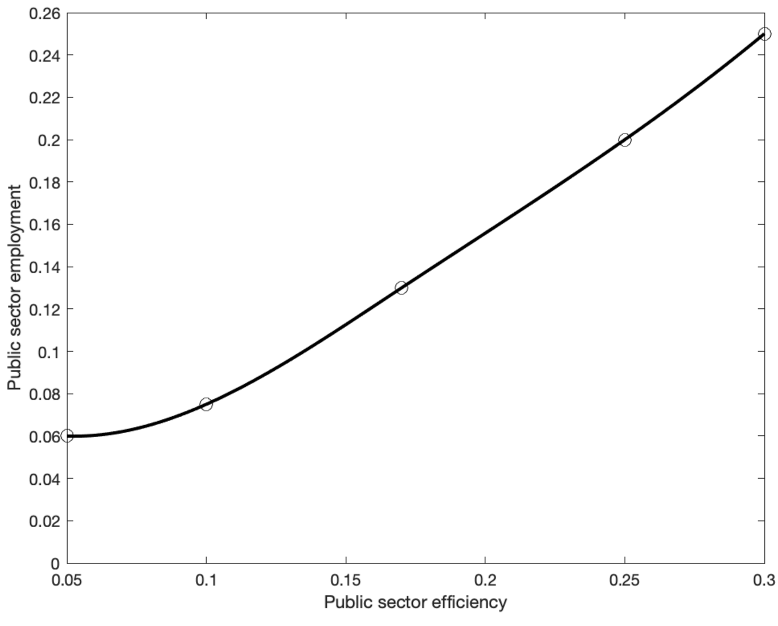
The bottom panel of Figure 6 depicts the optimal size of the public sector (across men and women) as a fraction of total employment. Consistent with our findings for the optimal premium, we observe that the higher the elasticity of private sector output to the public good, the higher the optimal share of employment allocated to the public sector. The fraction of public sector employment in our baseline economy (22%) is only optimal when the efficiency of the public sector is 0.3. For a value of γ of 0.1, the current size of the public sector is almost three times as large as the optimal.

Figure 6: Optimal average premium and optimal employment share in the public sector

(a) Optimal average premium



(b) Optimal public employment share



Notes: In each figure, circles correspond to values computed for specific values of γ, whereas the remaining points on the curve were interpolated. Public sector efficiency is proxied by γ. The baseline (calibrated) scenario corresponds to $\gamma = 0.3$. The optimal average public sector premium is defined as the ratio of the average wage in the public sector to the average wage in the private sector (minus one) that maximizes either q or TFP. The optimal average public sector employment share is defined as the ratio of employment in the public sector to total employment (minus one) that maximizes either q or TFP. Neither the optimal premium nor the optimal employment share differ significantly when q or TFP is maximized. Hence, the average of the premium and employment share implied by the maximization of q and the one implied by the maximization of TFP is displayed in the figure.

Figure 7 separates the optimal public sector premium and employment share by gender.

The figure shows that the optimal allocation of employment in the public sector rises for both men and women for higher levels of government efficiency, although the premium for men is practically negligible across the entire range of γ values under consideration. In panel (a), we observe that the optimal premium for women ranges from 19% for $\gamma = 0.05$ to 37% for $\gamma = 0.3$. In contrast, the optimal premium for men is close to zero regardless of the level of public sector efficiency levels (in the support under consideration). Thus, maximization of aggregate productivity in this economy (either output per worker q or TFP) requires paying a premium in the public sector to women but not to men. Similarly, panel (b) shows that the mass of women in the public sector as a share of total female employment ranges from 9% when the efficiency level of the public sector is low ($\gamma = 0.05$) to 40% when public sector efficiency is high ($\gamma = 0.3$). In the case of men, this share goes from 4% ($\gamma = 0.05$) to 18% ($\gamma = 0.3$). The fraction of employment optimally allocated to the public sector (adding both men and women) ranges from 6% under low levels of government efficiency ($\gamma = 0.05$) to 25% under high levels of efficiency ($\gamma = 0.3$).

In this economy, maximizing aggregate efficiency requires paying a premium in the public sector to women and not to men, and requires employing a comparatively higher share of women in the public sector compared to men. To understand this result, we consider a hypothetical scenario in which the premium is the same for women and men (that is, lower than the optimal for women, displayed in Figure A3 in the appendix, and higher than the optimal for men, displayed in Figure A4).[19] Such a wage structure would involve a larger number of women leaving the public sector to begin producing at home (as evident from Figure A3 when the line τ_f moves down). Conversely, in the case of men, a surge of entrepreneurs would enter the public sector (as depicted in Figure A4 when the line τ_m falls). A mass of women with mid-to-high skills would transition from the public sector into the home production sector, while a mass of men would transition from the entrepreneurial into the public sector. That is, offering a (roughly) equal premium for women and men in the public sector would prompt a significant portion of these women to engage in home production, while simultaneously reducing the size of the male entrepreneurial sector. Allocating more women into the public sector and away from the home production sector by increasing their premium, and allocating men into the

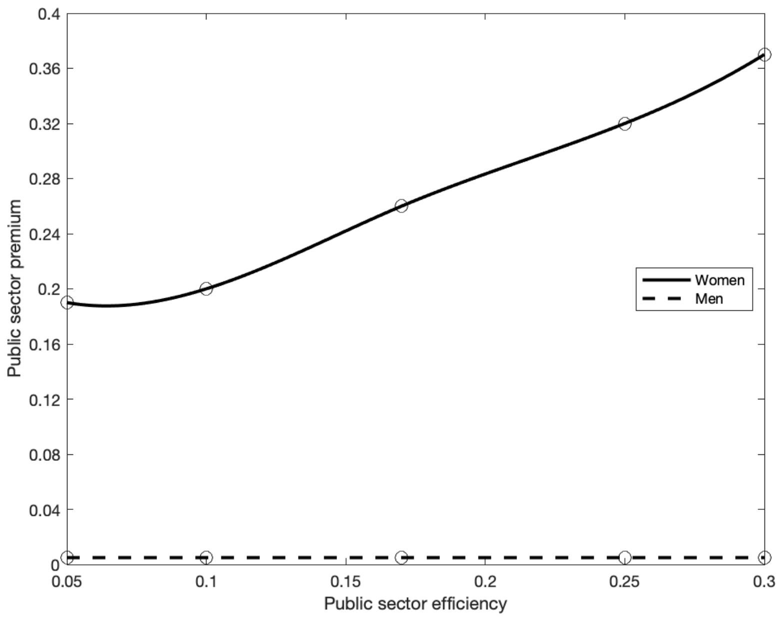
[19]Figures A1 and A2 int he appendix show the baseline (calibrated) allocation of female and male talent, respectively.

entrepreneurial sector and away from the public sector by lowering theirs, would then increase aggregate efficiency. The public sector premium at an optimal must therefore differ between men and women, with women in particular earning a higher premium.

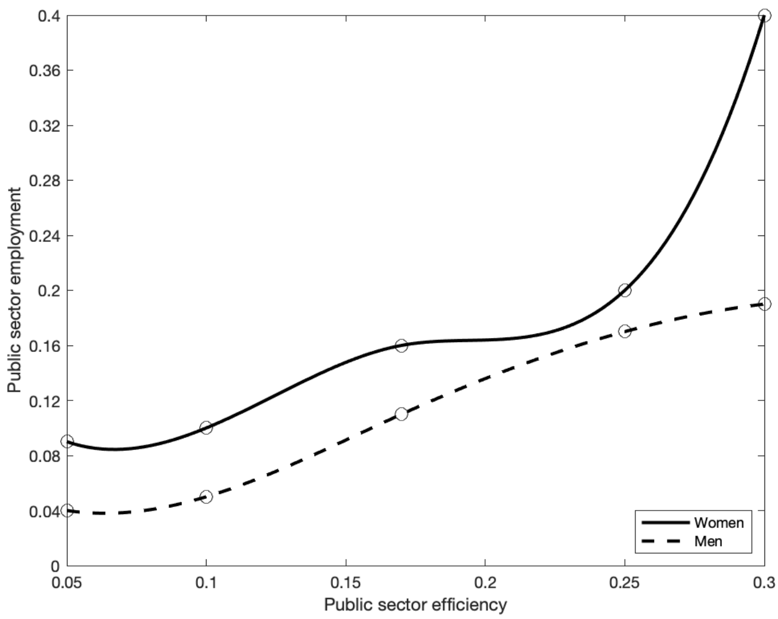
In Figure 8 we compute the efficiency gains achieved by adjusting the current public sector premium in our baseline economy to its optimal level for different values of γ. We show the two efficiency metrics (output per worker q and TFP). The figure shows that the current allocation of talent in our baseline economy is optimal (aggregate efficiency gains are minimal) when the value of γ (which is a proxy for the efficiency of the public sector in boosting private sector output) is relatively large (above 0.3). The lower the value of γ, the higher the efficiency gains in our baseline economy from reducing the current public sector premium and the current share of public sector employment (thus reallocating talent away from the public into the private sector).

Figure 7: Optimal average premium and optimal employment share in the public sector for men and women

(a) Optimal average premium for men and women

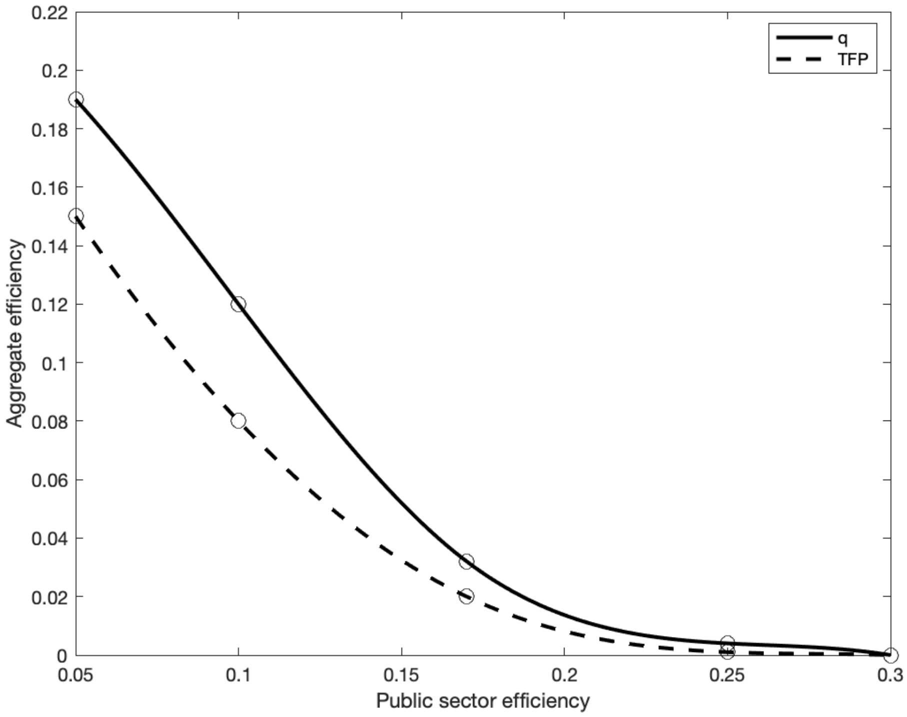


(b) Optimal public employment share among men and women



Notes: In each figure, circles correspond to values computed for specific values of γ, whereas the remaining points on the curve were interpolated. Public sector efficiency is proxied by γ. The baseline (calibrated) scenario corresponds to $\gamma = 0.3$. The optimal average public sector premium is defined as the ratio of the average wage in the public sector to the average wage in the private sector (minus one) that maximizes either q or TFP. The optimal average public sector employment share is defined as the ratio of employment in the public sector to total employment (minus one) that maximizes either q or TFP. Neither the optimal premium nor the optimal employment share differ significantly when q or TFP is maximized. Hence, the average of the premium and employment share implied by the maximization of q and the one implied by the maximization of TFP is displayed in the figure.

Figure 8: Aggregate efficiency gains



Note: Circles correspond to values computed for specific values of γ, whereas the remaining points on the curve were interpolated. The baseline (calibrated) scenario corresponds to $\gamma = 0.3$.

7 Concluding remarks

Extensive evidence shows that wages and certain non-pecuniary benefits favor public sector workers over those in the private sector. The existing literature often interprets this discrepancy as indicative of an overpaid public sector, conducting arbitrary counterfactual analyses to estimate aggregate gains from reducing the public sector premium to an optimal. Moreover, these frameworks discard, by design, the possibility for women of engaging in home production. Unlike these frameworks, the optimal public sector premium in our economy model is contingent upon the efficiency level of public spending. This novel feature allows us to perform non-arbitrary counterfactual analyses. The model also incorporates a home-production sector and the decision between market and non-market activities for women in our economy is therefore endogenous. Our findings underscore the importance of aligning the public sector premium with its optimal level, as evidenced by the substantial efficiency gains identified through our counterfactual exercises. Specifically, our calibration for Egypt shows that adjusting the public sector premium from the current to the optimal level, thus lowering the share of public sector employment to its optimal level, yields aggregate gains of 12% for output per worker and 8% for TFP under a conservative assumption for the level of efficiency of the public sector. The aggregate productivity gains are significantly larger for lower levels of the elasticity of private sector output to public goods.

We elucidate the drivers behind these aggregate productivity gains. In Egypt, an optimal talent allocation entails a reduction in the average public sector premium, albeit with differences between women and men. The optimal public sector premium remains positive for women but approaches zero for men. This optimal wage structure reduces the size of the public sector and encourages female entrepreneurial activities. Compared to a scenario where the premium is gender-neutral, it mitigates a significant transition of mid-high skilled women from the public sector to the home production sector and a contraction of the male entrepreneurial sector. Interestingly, we show that a lower female public sector premium increases female labor force participation in market activities.

Our analysis can be extended in several dimensions. For example, our framework could be leveraged to analyze the efficiency gains from public sector retrenchment in other regions of the world. We hope that the analysis conducted in this paper serves as a stepping stone to explore these or other research avenues.

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Appendix

A. Solution to the problem of an entrepreneur

The maximization problem of a type-i entrepreneur, for $i \in \{f, m\}$, with talent s is:

$$\max_{k_i, l_i^f, l_i^m} \pi_i(s) = y_i(s) - w_f l_i^f - w_m l_i^m - Rk_i. \tag{A1}$$

The first-order conditions are:

$$[k_i(s)] : \eta_e \alpha \frac{y_i(s)}{k_i(s)} = R, \tag{A2}$$

$$[l_i^f(s)] : \eta_e (1-\alpha) \left(\frac{y_i(s)}{l_i(s)}\right) \left(\frac{l_i(s)}{l_i^f(s)}\right)^{\frac{1}{\sigma}} = w^f, \tag{A3}$$

$$[l_i^m(s)] : \eta_e (1-\alpha) \left(\frac{y_i(s)}{l_i(s)}\right) \left(\frac{l_i(s)}{l_i^m(s)}\right)^{\frac{1}{\sigma}} = w^m. \tag{A4}$$

Then, using (A2) we get equation (5). In addition, from (A3) and (A4), we get:

$$l_i^f(s) = \left(\frac{\eta_e(1-\alpha)}{w^f} \frac{y_i(s)}{l_i(s)}\right)^{\sigma} l_i(s), \tag{A5}$$

$$l_i^m(s) = \left(\frac{\eta_e(1-\alpha)}{w^m} \frac{y_i(s)}{l_i(s)}\right)^{\sigma} l_i(s). \tag{A6}$$

Plugging (A5) and (A6) in (2), we get equation (6). Then, using (5) and (6) in (1), we get equation (4).

Finally, from (A3) and (A4), we get:

$$\frac{l_i^m(s)}{l_i^f(s)} = \left(\frac{w^f}{w^m}\right)^{\sigma}, \tag{A7}$$

and from (2), we get:

$$\frac{l_i(s)}{l_i^f(s)} = \left(1 + \left(\frac{l_i^m(s)}{l_i^f(s)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{A8}$$

$$\frac{l_i(s)}{l_i^m(s)} = \left(1 + \left(\frac{l_i^f(s)}{l_i^m(s)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A9}$$

Plugging (A7) into (A8) and (A9), we get equations (7) and (8).

B. Solution to the problem of the public firm

The maximization problem of the public firm is:

$$\max_{K_p, L_p^f, L_p^m} B = G(Z_p) - (1+\tau_f)\, w_f \tilde{L}_p^f - (1+\tau_m)\, w_m \tilde{L}_p^m - w_f L_p^f - w_m L_p^m - RK_p. \tag{B1}$$

The first-order conditions are:

$$[K_p(Z_p)] : \eta_g \kappa \frac{G(Z_p)}{K_p(Z_p)} = R, \tag{B2}$$

$$[L_p^f(Z_p)] : \eta_g (1-\kappa) \left(\frac{G(Z_p)}{L_p(Z_p)}\right) \left(\frac{L_p(Z_p)}{L_p^f(Z_p)}\right)^{\frac{1}{\sigma}} = w^f, \tag{B3}$$

$$[L_p^m(Z_p)] : \eta_g (1-\kappa) \left(\frac{G(Z_p)}{L_p(Z_p)}\right) \left(\frac{L_p(Z_p)}{L_p^m(Z_p)}\right)^{\frac{1}{\sigma}} = w^m. \tag{B4}$$

Then, using (B2) we get equation (16).Then, using (B3) and (B4), we derive:

$$L_i^f(Z_p) = \left(\frac{\eta_g(1-\kappa)}{w^f} \frac{G(Z_p)}{L_p(Z_p)}\right)^{\sigma} L_p(Z_p), \tag{B5}$$

$$L_i^m(Z_p) = \left(\frac{\eta_g(1-\kappa)}{w^m} \frac{G(Z_p)}{L_p(Z_p)}\right)^{\sigma} L_p(Z_p). \tag{B6}$$

Plugging (B5) and (B6) in (12), we get equation (6). In addiiton, using (16) and (17) in (11), we get equation (15).

Finally, from (B3) and (B4), we get:

$$\frac{L_p^m(Z_p)}{L_p^f(Z_p)} = \left(\frac{w^f}{w^m}\right)^{\sigma}, \tag{B7}$$

and from (12), we derive:

$$\frac{L_p(Z_p)}{L_p^f(Z_p)} = \left(1 + \left(\frac{L_p^m(Z_p)}{L_p^f(Z_p)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{B8}$$

$$\frac{L_p(Z_p)}{L_p^m(Z_p)} = \left(1 + \left(\frac{L_p^f(Z_p)}{L_p^m(Z_p)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{B9}$$

Then, plugging (B7) into (B8) and (B9), we get equations (18) and (19).

C. Euler equation

The representative household gets flow utility from a sequence of per capita consumption, $\{c_t\}_{t=0}^{\infty}$, and maximizes:

$$U(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{C1}$$

where $\beta \in (0,1)$ is the discount factor.

The representative household takes as given all prices and the occupational choices of their member and chooses a sequence of per capita consumption and aggregate capital to solve

$$\max_{\{c_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t \ln c_t \tag{C2}$$

$$\text{s.t. } c_t + K_{t+1} = W_t + R_t K_t + (1-\delta) K_t,$$

$$K_0 > 0, L_0 > 0,$$

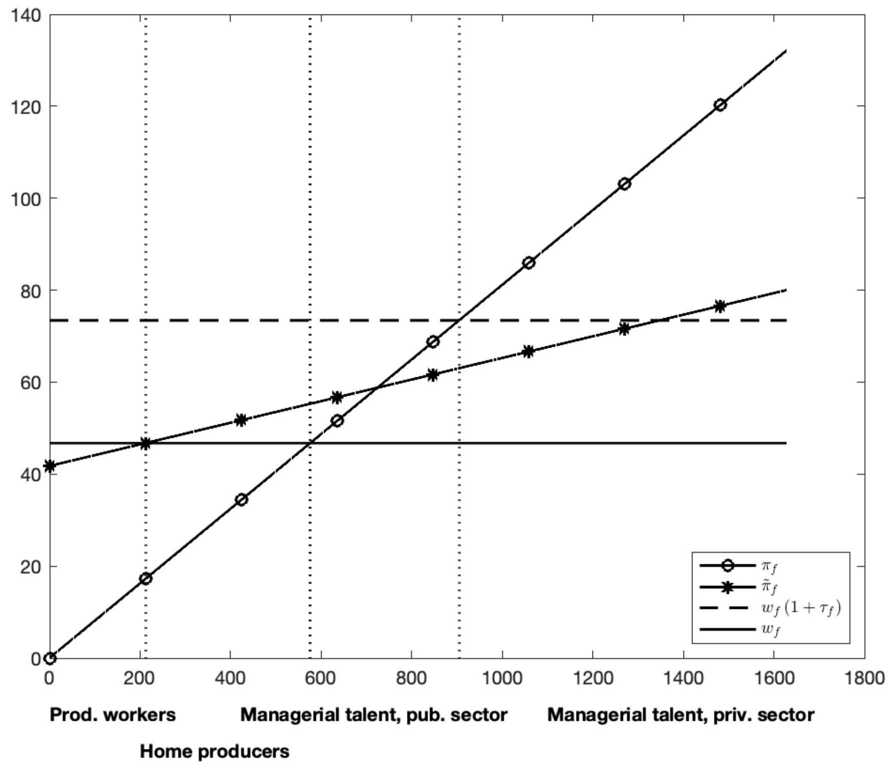
The first order conditions for per capita consumption and aggregate capita implies

$$\frac{c_{t+1}}{c_t} = \beta(R_{t+1} + 1 - \delta) \quad \text{for all } t \geq 0, \tag{C3}$$

Equation (C3) is the standard Euler equation. In steady-state, we have: $R = 1/\beta + \delta - 1$.

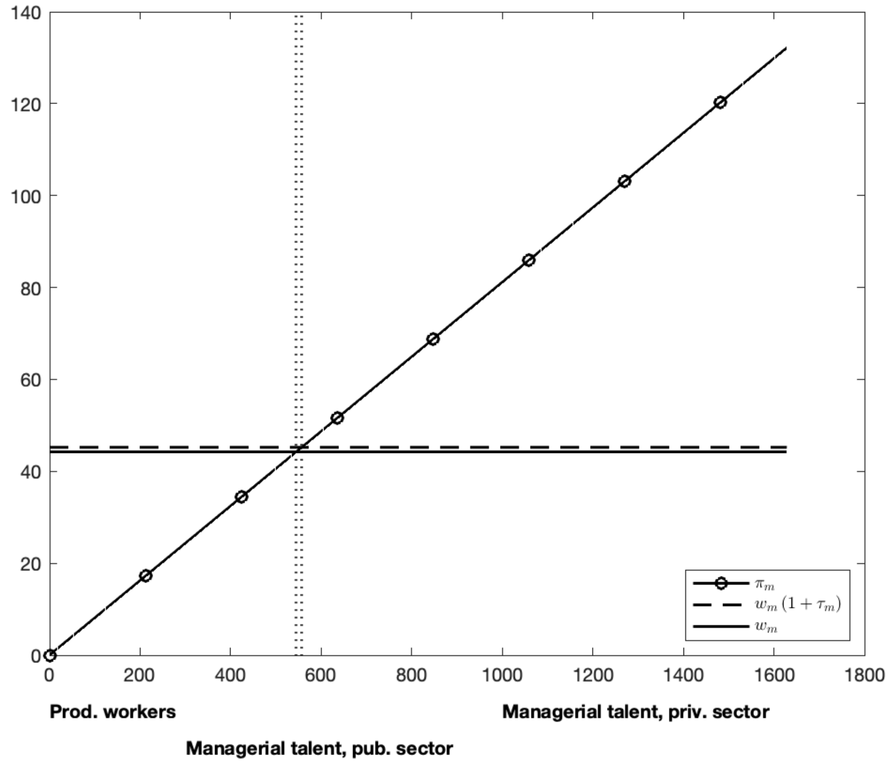
D. Additional figures

Figure A1: Equilibrium allocation of female talent, baseline



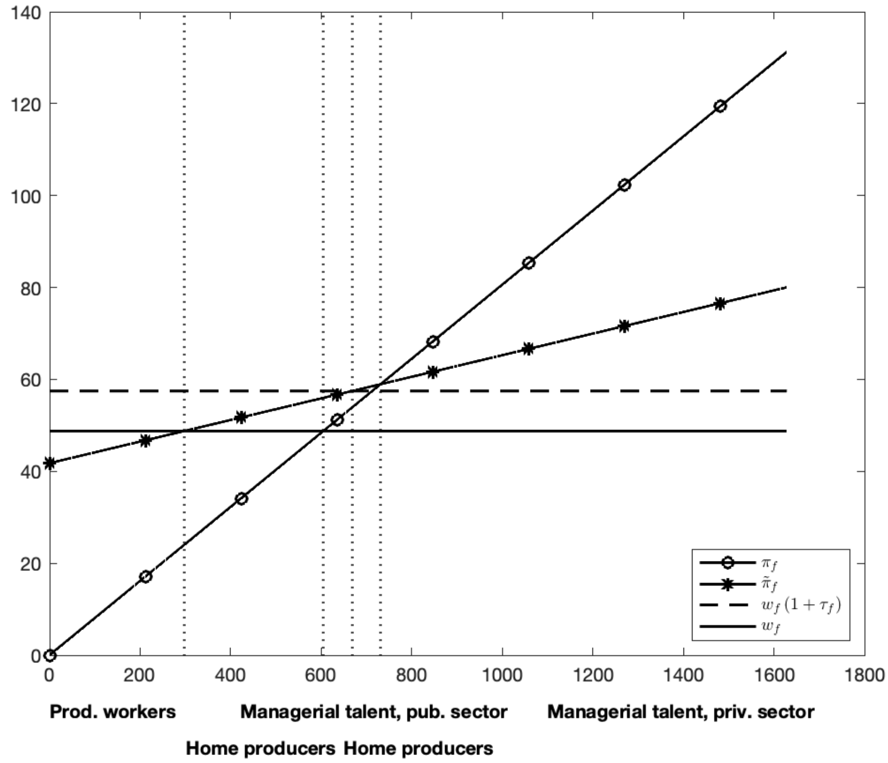
Note: π_f is the profits accrued by entrepreneurs in the private sector; $\tilde{\pi}_f$ is the net value of home production; w is the wage paid to production labor; τ_f is the public sector premium. The baseline scenario corresponds to the calibrated premium.

Figure A2: Equilibrium allocation of male talent, baseline



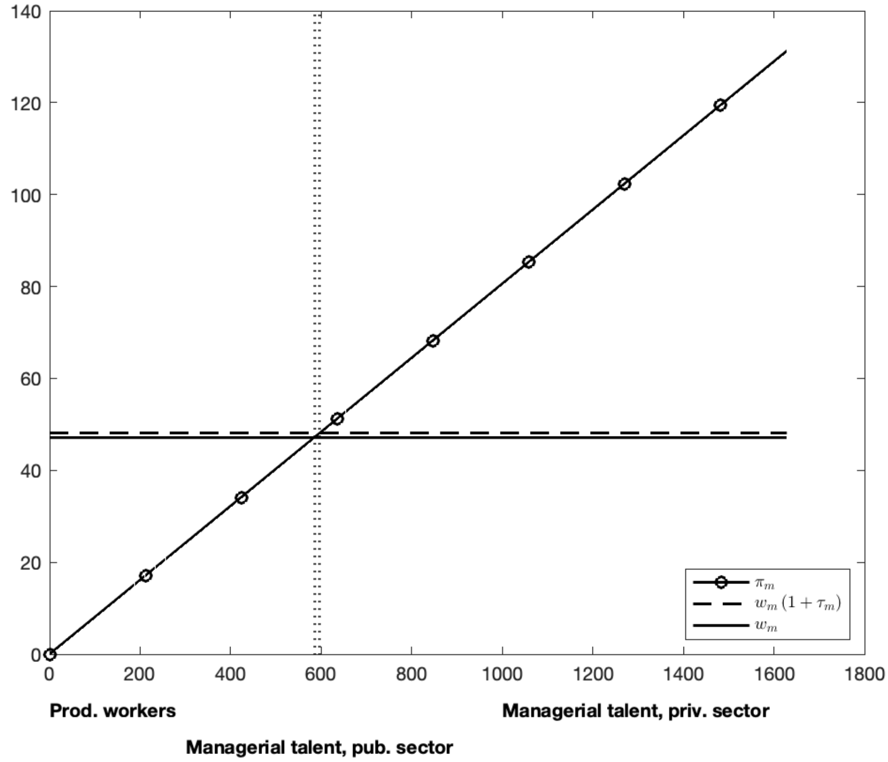
Note: π_m is the profits accrued by entrepreneurs in the private sector; w is the wage paid to production labor; τ_m is the public sector premium. The baseline scenario corresponds to the calibrated premium.

Figure A3: Optimal allocation of female talent, counterfactual



Note: π_f is the profits accrued by entrepreneurs in the private sector; $\tilde{\pi}_f$ are the net value of home production; w is the wage paid to production labor; τ_f is the public sector premium. The counterfactual scenario corresponds to the optimal premium for $\gamma = 0.1$.

Figure A4: Optimal allocation of male talent, counterfactual



Note: π_m is the profits accrued by entrepreneurs in the private sector; w is the wage paid to production labor; τ_m is the public sector premium. The counterfactual scenario corresponds to the optimal premium for $\gamma = 0.1$.